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PROOF THAT NO TWO DIFFERENT ELLIPSES CAN BE PARALLEL.

PY PROF. E. W. HYDE, CINCINNATI, OHIO.

The equation of an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

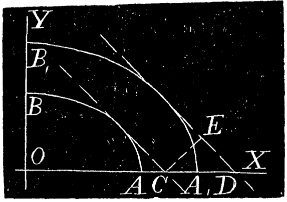
Therefore, if there can be a parallel ellipse its equation must be of the form

$$\frac{x^2}{(a+c)^2} + \frac{y^2}{(b+c)^2} = 1;$$

i. e. the same quantity c is added to each semi-axis. Now if these curves are parallel the distance between a tangent to one and a parallel tangent to the other on the same side of the origin must be equal to c. The equations of a pair of such parallel tangents are

(1) $y = mx + \sqrt{(a^2m^2 + b^2)}$ and

(2) $y = mx + \sqrt{[(a+c)^2m^2 + (b+c)^2]}$.



The distance between the two tangents in the figure is

$$CE = CD\sin(\tan^{-1}m) = (OD - OC)\frac{m}{\sqrt{(1+m^2)}}$$

For convenience let the tangents be inclined at an angle of 135°, i. e. let $m = -1$; then by making $y = 0$ in (1) and (2) $OC = \sqrt{(a^2 + b^2)}$

and $OD = \sqrt{[(a+c)^2 + (b+c)^2]}$.

$$\therefore \quad CE = \sqrt{\tfrac{1}{2}} \times \{ \sqrt{[(a+c)^2 + (b+c)^2]} - \sqrt{(a^2+b^2)} \}.$$

When $b = a$, $CE = c$ as it should; but if b is different in value from a, then CE will be different in value from c, and the two curves cannot be parallel. To prove this, place

$$\sqrt{\tfrac{1}{2}} \times \{ \sqrt{[(a+c)^2 + (b+c)^2]} - \sqrt{(a^2+b^2)} \} = c.$$

Transposing, squaring and cancelling we obtain finally $a = b$, showing that this condition is *necessary* in order that the equation may be true.

[We have also received a paper from Prof. Hyde supplimentary to his solution of Mr. Church's problem, (p. 76), in which he effects a solution of the last part of that problem, or rather of the "secondary" problem to which he has reduced that problem by projection, in a more simple and elegant manner. We have also received solutions of the same problem from Prof. Johnson and Dr. Eggers which we hope, at some future time, to present to our readers.—Ed.]